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Energy (Adjacency Energy) of a 3-uniform T_2 Hypergraph

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ABSTRACT

Let H be a 3-uniform T_2 Hypergraph of order $n \geq 5$. The adjacency matrix of the 3-uniform T_2 Hypergraph is defined by A(H) = $\begin{cases} \left|\{D_k \in D : (x_i, x_j) \subseteq D_k\right| & if\ x_i \neq x_j \\ 0\ otherwise \end{cases}$. The adjacency energy of a 3-uniform T_2 Hypergraph is the sum of the absolute eigenvalues of its adjacency matrix. In H, AE(H) $\geq \Delta^2 + \delta^2 + \frac{1}{\Delta} + \frac{\delta}{\Delta^2}$, equality holds only if n = 11 in H.

Keywords: *T_2 Hypergraph, 3-uniform T_2 Hypergraph, adjacency matrix, adjacency energy.*
Subject Classification: *05C65*

1 Introduction

The basic definitions and terminologies of a hypergraph are not given here; we refer it [1] and [2]. The concept of a Hypergraph was introduced by Berge in 1967. Later, different authors studied the same concept in [3] and [4]. Seena V and Raji Pilakkat were introduced to Hausdorff Hypergraph, T_0 Hypergraph and T_1 Hypergraph. Based on [5], [6] and [7] we introduced a new class of Hypergraph namely T_2 Hypergraph, 3-uniform T_2 Hypergraph and the parameter adjacency energy is studied for the same. Throughout this article, H is a simple connected 3-uniform T_2 Hypergraph with order n and size m. Here the order and size are the minimum numbers of vertices and edges used to define a 3-uniform T_2 Hypergraph. In A(H), λ_1 is the largest eigenvalue and λ_n is the smallest eigenvalue. The following definitions and theorems are used in the sequel.

Definition 1.1. [5] A hypergraph H = (X, D) is said to be a Hausdorff hypergraph if for any two distinct vertices u, v of X there exists hypergraph D_1 and D_2 such that u $\in D_1$, v $\in D_2$ and $D_1 \cap D_2 = \emptyset$.

Definition 1.2. [6] A hypergraph H = (X, D) is said to be a T_0 Hypergraph if for any two distinct vertices u, v of X there exists a hyperedge containing one of them but not the other.

Definition 1.3. [7] A hypergraph H = (X, D) is said to be a T_1 Hypergraph if for any two distinct vertices u, v of X there exists a hyperedge containing u but not v and another hyperedge containing v but not u.

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Definition 1.4. [8] The adjacency matrix is the square matrix in which rows and columns are indexed by the vertices of H and where for all u, v ∈ X, u ≠ v, $a_{uv} = |\{d \in D / u, v \in D\}|$ and $a_{uu} = 0$.

Definition 1.5. [8] The adjacency energy of a hypergraph is the sum of the eigenvalues of its adjacency matrix.

Definition 1.6. [9] A hypergraph H = (X, D) is said to be a T_2 Hypergraph if for any three distinct vertices u, v and w in X there exists a hyperedge containing u and v but not w and another hyperedge containing w but not u and v.

Definition 1.7. A T_2 Hypergraph H = (X, D) is said to be a 3-uniform T_2 Hypergraph if every hyperedge contains exactly three vertices.

2 Adjacency matrix and energy of a 3-uniform T_2 Hypergraph

In this section, we find the energy of a 3-uniform T_2 Hypergraph H, using adjacency matrix. Consider a 3-uniform T_2 Hypergraph H given in Figure 1 with 8 vertices and 7 edges.

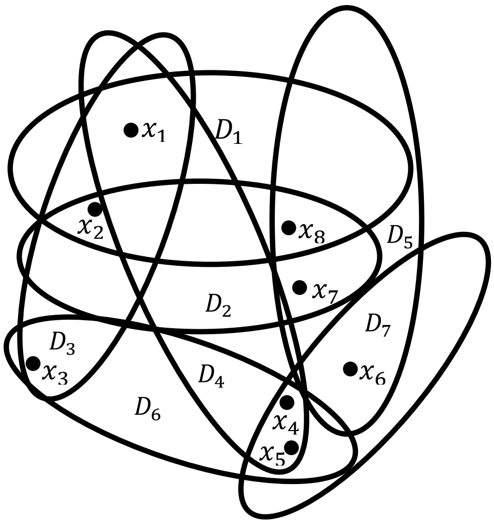

Figure 1: 3-uniform T_2 Hypergraph

The corresponding adjacency matrix of H is given by

$$A(H) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 & 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The adjacency eigenvalues of A (H) are λ = 5.42, 3.31, 1.43, – 1.408, –2.298, – 2.588, – 3.

Therefore, the adjacency energy AE(H) = 20.314.

Result 2.1. Let H be a 3-uniform T_2 Hypergraph with n ≥ 6. Then the adjacency energy AE(H) ≥ $\Delta^2 + \delta^2 + \frac{1}{\Delta} + \frac{\delta}{\Delta^2}$, equality holds for n = 11. The below table 1, shows the adjacency energy of a 3-uniform T_2 hypergraph for its order for n ≥ 6.

Number of Vertices	AE(H)	$\Delta^2 + \delta^2 + \frac{1}{\Delta} + \frac{\delta}{\Delta^2}$
6	10.28	4.32
7	15.2	13.55
8	20.314	13.55
9	23.51	20.32
10	26.04	25.44
11	34.32	34.32
…	…	…
n	…	$\Delta^2 + \delta^2 + \frac{1}{\Delta} + \frac{\delta}{\Delta^2}$

Table 1: Adjacency energy of a 3-uniform T_2 Hypergraph

Clearly from Table 1, we can identify that, AE(H) ≥ $\Delta^2 + \delta^2 + \frac{1}{\Delta} + \frac{\delta}{\Delta^2}$ for n ≥ 6.

Result 2.2. Let H be a 3-uniform T_2 Hypergraph with n ≥ 5. Then $\lceil \lambda_1 \rceil = \Delta + \delta$. From the following Table 2, we can easily verify the result.

Number of Vertices	λ_1	$\lceil \lambda_1 \rceil$	$\Delta + \delta$
5	5.78	6	6
6	4.09	4	4
7	4.87	5	5
8	5.42	6	6
9	6.15	6	6
10	6.68	7	7
11	8.27	8	8
…	…	…	…
n	…	…	$\Delta + \delta$

Table 2: Values of $\Delta + \delta$

Result 2.3. Let H be a 3-uniform T_2 Hypergraph with n ≥ 5. Then $\lceil \lambda_1 \rceil \geq \lfloor \sqrt{n} + k \rfloor$, equality holds for n = 6 and 7.

Number of Vertices	λ_1	$\sqrt{n} + k$	$\lceil \lambda_1 \rceil$	$\lfloor \sqrt{n} + k \rfloor$
5	5.77	5.23	6	5
6	4.09	5.45	5	5
7	4.87	5.65	5	5
8	5.42	5.83	6	5
9	6.15	6	7	6
10	6.68	6.16	7	6
11	8.21	6.32	9	6
…	…	…	…	…
n	…	…	…	$\lfloor \sqrt{n} + k \rfloor$

Table 3: Values of $\lceil \lambda_1 \rceil$

Result 2.4. Let H be a 3-uniform T_2 Hypergraph with n ≥ 5. Then $\left\lceil (\det A(H))^{\frac{1}{n}} \right\rceil = \left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$

Number of Vertices	$(\det A(H))^{\frac{1}{n}}$	$\left\lceil (\det A(H))^{\frac{1}{n}} \right\rceil$	$\sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}}$	$\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$
5	1.15	2	1.12	2
6	1.59	2	1.55	2
7	1.87	2	1.66	2
8	1.92	2	1.75	2
9	1.86	2	1.84	2
10	1.96	2	1.93	2
11	2.62	3	2.02	3
…	…	…	…	…
n	…	…	…	$\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$

Table 4: Values of $(\det A(H))^{\frac{1}{n}}$

Theorem 2.5. Let H be a 3- uniform T_2 hypergraph with n≥ 5. Then AE(H) < $\frac{n(\Delta+\delta)^2}{\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil}$

Proof. We have, $[\lambda_1] > \left\lceil \left(detAE(H)\right)^{\frac{1}{n}} \right\rceil = \left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$

$$[\lambda_1] \sum_{i=1}^{n} |\lambda_i| > \left\lceil \left(detAE(H)\right)^{\frac{1}{n}} \right\rceil \sum_{i=1}^{n} |\lambda_i| = \left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil \sum_{i=1}^{n} |\lambda_i|$$

Since, $[\lambda_1] = \Delta + \delta > |\lambda_i| \; \forall \; i = 2,3, \dots, n.$

Therefore, $n[\lambda_1]^2 = n(\Delta+\delta)^2 > \left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$ AE(H)

Hence, AE(H) < $\frac{n(\Delta+\delta)^2}{\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil}$

Illustration 2.6. Consider a 3-uniform T_2 hypergraph with n = 8. From the Table 1, Table 2 and Table 4, AE (H) = 20.31, $\Delta + \delta = 6, \left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil 2.$ Here, AE(H) = 20.31 < 144 $= \frac{n(\Delta+\delta)^2}{\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil} = \frac{8\times 36}{2}$.

Theorem 2.7. Let H be a 3- uniform T_2 hypergraph with n ≥ 5. Then

AE(H) < $\lceil \lambda_1 \rceil$ + $\frac{(n-1)(\lfloor \sqrt{n}+k \rfloor)^2}{\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil}$.

Proof. In H, $\lceil \lambda_1 \rceil \geq \lfloor \sqrt{n}+k \rfloor$ > $\left\lceil \left(detAE(H)\right)^{\frac{1}{n}} \right\rceil$ = $\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$

$\lfloor \sqrt{n}+k \rfloor \sum_{i=2}^{n} |\lambda_i|$ > $\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil \sum_{i=2}^{n} |\lambda_i|$

Since $\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$ > $|\lambda_i| \; \forall \; i = 2,3,\ldots,n.$

(n – 1)($\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil^2$) > $\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$ (AE(H) – $|\lambda_1|$)

> $\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil$(AE(H) – $\lceil \lambda_1 \rceil$)

Hence, AE(H) < $\lceil \lambda_1 \rceil$ + $\frac{(n-1)(\lfloor \sqrt{n}+k \rfloor)^2}{\left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil}$.

Illustration 2.8. Consider a 3-uniform T_2 hypergraph with n = 8. From the Table 1, Table 3 and Table 4, AE (H) = 20.31, $\lfloor \sqrt{n}+k \rfloor = 5, |\lambda_1 = 6|, \left\lceil \sqrt{\frac{n+\sqrt{\frac{k}{2}}}{k}} \right\rceil = 2.$ Here, AE(H) = 20.31 < $92.5 = \frac{7\times 25}{2} + 5.$ Hence the above theorem is verified.

3 Conclusion

In this article, we studied the adjacency matrix and its energy for a 3-uniform T_2 hypergraph. Also, we established the bounds of the adjacency energy of the 3-uniform T_2 hypergraph using various graph parameters.

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